Wolf, Goat, and Cabbage: An Analysis of Students' Roles and Cognitive and Metacognitive Behaviors in Small Group Collaborative Problem-Solving

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ABSTRACT: The research reported on in this paper examines students' cognitive and metacognitive behaviors as they are manifested in non-routine problem-solving in small groups in a mathematics classroom. Students' cognitive and metacognitive moves are analyzed as well as the connection between the interaction of both cognitive and metacognitive processes and the effectiveness of the problem-solving process. Some links between modes of discourse, modes of thought in social interactions in small groups, group communicative patterns, and the interplay among students' roles within the groups are investigated, in order to identify factors which influence the process of collaborative problem-solving, and which contribute to optimal and successful cooperative work in group settings. The results of the study indicate that a setting with a balance between the incidence of cognitive and metacognitive behaviors of the participants and relatively equal participation of all the members of the group, without pronounced patterns of domination and with a high degree of tolerance of other's opinions, are factors which contribute to successful work in group settings.

Problem solving is one of the most important aspects of doing mathematics, and is considered to be a crucial domain of mathematical learning and knowing (NCTM, 1989). Research findings (Schoenfeld, 1985; Forman & McPhail, 1993; Cobb, Wood & Yackel, 1993; Cole & Nicolopoulou, 1993; Cai, Mamona-Downs, Weber, 2005) indicate that current interpretations of problem solving and of the factors that contribute to its success might be too narrow. Several studies investigating mathematical problem-solving (Garofalo & Lester, 1985; Schoenfeld, 1985; Lambdin,1993, Artzt & Armour-Thomas, 1992; Muir, Beswick, Williamson, 2008) suggest that success in the latter may depend on the active monitoring and regulation of the cognitive processes engaged in the larger process itself.

However, if we accept the assumption that students' knowledge is individually constructed, and that success in problem-solving is an independent achievement, there is the risk of ignoring the role of the other in the process. Drawing on Lev Vygotsky's theory, which exerted a strong influence on western scholars, current researchers and educators operate with a multi-layered model of the relationship between cognitive, psychological and sociocultural factors in the learning process. Such a model also suggests that the kinds of activities in which students engage in their everyday classroom life have a profound impact on the development of their cognitive, metacognitive and communicative functions. If this is the case, it follows that systematic analysis of collaborative classroom problem-solving activities can help teachers identify, not just levels of development in students' patterns of communication, goal-setting, social interaction, and ability to solve particular problems, but how and under what influences they change. Moreover, it is important to know how all those factors may communicate in ways that either promote or impede successful problem solving. Once this becomes clearer, it might be easier for teachers to identify the cognitive and communicative opportunities that are available in the microculture of the classroom.

Purposes of the Research

Given the multiple dimensions sketched above, the goal of the mini-research project described here was manifold. First, I intended to analyze students' cognitive and metacognitive behaviors (moves) in the solution of non-routine problems that require more than one step--to investigate how their choice and management of strategies can be responsible for successful problem-solving. Students' behaviors exhibited during problem solving such as reading the problem or listening are characterized here as cognitive. When behaviors are accompanied by self-regulation and self-monitoring---for example, such behaviors may be exhibited in implementation of a plan for solving a math problem or verification of a solution----they can be characterized as metacognitive.

Given the sociocultural dimension, I also examined the phenomenon of cognitive processing as it evolves while students are engaged in problem-solving in small groups, with special attention to the observable links between modes of discourse and modes of cognitive and metacognitive behaviors during social interactions in small groups.

Literature Review

Solving non-routine problems requires thoughtful analysis: defining the problem, planning a strategy for its solution, implementing the strategy, and checking the results (Polya, 1973). A thoughtful analysis entails understanding, problem solving skills, and motivation. Effective problem solving also requires cognitive flexibility (Baroody, 1987). According to Schoenfeld (1985) mathematical problem-solving requires, not just a basis in mathematical knowledge, but that adequate resources are made available to the individual in the given context. Heuristic efficiency in problem-solving, he claims, might depend heavily on managing the resources available and the execution of control within the environment of the given problem.

The key component in Schoenfeld's model of problem-solving proficiency is the capacity to monitor the state of the solution as it evolves, and to rethink each new move in light of one's emergent understanding of the problem and its constraints. He argues that "resources and control are two qualitatively different, though deeply intertwined, aspects of mathematical behavior" (p.135). Recent research tends to support Schoenfeld's view, and indicates that performance on a task is positively correlated with one's mathematical knowledge, on the degree of one's metaknowledge, and on the interaction between the two (Lambdin, 1993; Artzt & Armour- Thomas, 1992; Carpenter & Fennema, 1992; Carpenter & Fennema, 1996; Muir et al, 2008). Furthermore, Vygotsky's theory offers an approach to the study of cognition in social contexts, and explains the development of mental functions as a result of the child's interaction with adults and peers. Vygotsky claims that "every function in the child's cultural development appear twice, on two levels: first on the social and later on the psychological level—first between people as an interpsychological category and then within the learner as an intrapsychological category (Vygotsky, 1978, p.128). Many authors (Bruner, 1986; Minick, 1989; Wertsch, 1985) suggest further that the Vygotskyan perspective on teaching and learning processes goes far beyond the mere transfer of knowledge from teacher to learner. Collaborative problem-solving activities understood from a Vygotskyan perspective have drawn increasing attention from researchers. In fact recent studies indicate that they provide a context in which additional supports for, as well as challenges to, students' thinking occur (Cobb, 1995; Yackel, 1995; Lampert, 1990). School activities that involve collaborative problem-solving provide a rich environment for students to learn how to work together with complex tasks. Many studies discuss the social, affective and cognitive benefits of cooperative learning and cooperative problem-solving, but little attention has been paid to how students working collaboratively negotiate goals and define tasks, and how they carry them out using shared means of communication.

Schoenfeld's method of protocol analysis focuses on decision-making at the executive or control level of the problem-solving procedure, and suggests a way to examine individual behavior throughout the evolution of the solution. In his model, protocols are partitioned into sections called "episodes." Each episode represents a period of time during which an individual or a group of problem solvers is engaged in one large task and are consistently displaying one form of behavior. The junctures between episodes are the points at which the direction of the problem-solving process changes significantly. Schoenfeld identifies six characteristic behaviors which he claims to find in sequence in the problem-solving process: "reading" the problem, analysis, exploration, planning, implementation and verification. His protocol analysis offers a picture of executive decision-making, control level, and management of the problem-solving process. Schoenfeld has not specified a way to determine the cognitive level of each episode, but other research studies (Schoenfeld, 1985; Lambdin, 1993; Artzt & Armour-Thomas, 1992) indicate that the reliability of the parsing protocols analysis is substantial. Nor has Schoenfeld fully addressed the collaborative aspects of problem-solving, but Lambdin (1993) uses parsing protocols to analyze cooperative mathematical problem-solving in small groups. She adds to her analysis of the procedure itself a further analysis of the monitoring moves and the roles characteristically played by individuals working together, which gives a more detailed picture of the dynamics of the process and the cognitive levels of the moves. In addition, Artzt and Armour-Thomas (1992) use parsing protocols to delineate the types and levels of the cognitive processes individuals use as they work together in a small group setting, and to understand how these processes affect the entire process of solving a given problem. Their study suggests that a certain balance between cognitive and metacognitive processes within a group is required to ensure the success of problem solving efforts.

Design of the Study

In order to analyze the data I collected, I utilized transcripts of students' discussions in five small groups during mathematical problem-solving events. For more in-depth analysis, I also used my observations of these same students over a one-month period, and information given by their teacher. During observations of classes prior to the recorded discussions, I noted patterns of math-resistant behavior – especially toward some geometry and algebraic problems. Students appeared to see no connection between these problems and "real life," and understood them as impractical and useless. Thus, I suggested to the teacher that we try a problem that seemed to me to be more connected to their informal knowledge, hoping that it would draw their attention, and lead them more easily into collaborative work. The problem given was: A wolf, a goat, and a cabbage are on one side of a river. A boatman is given the task of transporting them to the other side, but there are two conditions: first, he can only take one at a time; and second, the wolf must not be left alone with the goat and the goat must not be left alone with the cabbage. Find a way for the boatman to carry all three across the river.

The site of the study was a magnet high school in New Jersey, founded one year previously and operated under the administration of a college of education of a university nearby. When I recorded the discussions for analysis, I had already been visiting two tenth grade geometry sections for a month, and meeting regularly with the teacher of both sections to discuss students' learning problems, the psychological environment, specific instructional strategies, and aspects of classroom dynamics. The majority of students were of African-American and Latino ethnic origin – a total of 30 tenth-grade students, 13 boys and 17 girls, who comprised two classes taught by one teacher. On the teacher's account, they had little experience in working in small groups. Typically, the whole class worked together, and the last minutes of the class period were spent working in pairs. Since they rarely changed their seats, the working pairs – chosen by student preference – were virtually permanent. Thus, students had no experience working with different students, either individually or in groups. For the purpose of this study the teacher formed new groups of two, three or four students of differing abilities in mathematics. I was given permission by the principle and the teacher to make a 45 minute tape of student discussions as they engaged in the process of solving non-routine problems. On the day on which I recorded, the teacher formed four groups during the second period and five groups during the third, and asked each to solve the problem I had chosen, stating that they would be working collaboratively. I audiotaped two groups who worked during the second period and three groups during the third, all of them randomly selected. The resulting tape was transcribed and used as data for analysis.

I analyzed each discussion using Schoenfeld's six-step protocol, and made a description and analysis of each student's role in the problem solving process. Using a framework developed by Artzt & Armour-Thomas (1993) I coded students' behaviors as cognitive and/or metacognitive, and examined the way the two interact and affect the outcome of mathematical problem-solving in small groups. Finally, I examined the structures and dynamics of students' cognitive and metacognitive processes. I tried to articulate the complexity of the problem-solving process accurately, taking into account the influence of social, psychological and cognitive factors. In addition to using Schoenfeld's parsing protocol as an analytic tool, I included an analysis of subjects' regulatory and monitoring roles. In order to analyze the cognitive and metacognitive behaviors of the subjects while they worked together in small groups, I adopted Artzt & Thomas's framework, which is derived from Schoenfeld's model, and expands the latter's episodic categories for coding students behaviors with two new categories – understanding the problem and watching/listening – given that, quite often, students made comments about the conditions of the problem, or tried to clarify their understanding, and by definition collaborative work implies that at certain moments students watch and listen to others.

Following Artzt and Thomas, I classified each of the eight episodes as cognitive or metacognitive. In keeping with the working definition developed by Garofalo & Lester (1985, p.164), I identified behaviors as predominantly cognitive when actions were made toward further processing – not including monitoring or regulation – and as metacognitive when actions indicated procedural assessment, state assessment (i.e. assessment of the current status of the problem-solving process), self-regulation, and partner reflections (Lambdin, 1993). The "reading" episode is assigned cognitive characteristics. Analyzing and planning and understanding are assigned predominately metacognitive characteristics, because they require reflective thinking and regulatory functioning to keep the process within the frame of the problem as defined. Exploring and verifying episodes can be assigned either cognitive or metacognitive characteristics, depending on whether the behavior is guided by monitoring. Schoenfeld (1987, p.194) drew the conclusion, based on the bulk of his research studies, that in many cases exploration often results in "wild goose chases." When exploration, implementation and verification are accompanied by self-regulation and self-monitoring, such behaviors can be categorized as metacognitive – otherwise they should be assigned cognitive characteristics. Watching/listening is not categorized as either cognitive or metacognitive, because it offers no evidence that makes it possible to infer a level of cognition.

Transcripts were analyzed by one-minute intervals, after which I coded the heuristic episode and the cognitive level that best represented the students' behaviors exhibited during the interval under examination. The behaviors were listed in sequence, then were categorized in two ways – by episode and by cognitive level. Charting each student's behavior in this way created an individual profile for each student. By counting the number of cognitive and metacognitive behaviors and dividing them by the total number of behaviors coded in the group, a profile was obtained for each group member's cognitive and metacognitive participation and contribution within the group. Space limitations do not allow for the inclusion of the protocol analysis and the behavior analysis of all five groups in this paper, and I have therefore chosen to present analyses only for Groups 1, 2, and 4. What follows are three representative examples of three of the protocol analyses of the groups involved, and a discussion which draws implications from all five.

Analysis

Protocol Analysis Group 1

The first discussion took place among four students, A, B, C, D. The students read the problem silently. There were no comments or attempts to define a goal. They moved immediately into trial-and-error exploration, which began in the pursuit of two ideas, within the context of their professed intention to explore the possibility of solving a part of the problem – i.e., bringing one of the items to the other side of the river. The first idea led them to the conclusion that they couldn't proceed further without violating the conditions of the problem. Their second idea was immediately assessed as unworkable, considering the problem constraints. During the second minute student B – who, based on the comparative number and length of her interventions, of which a comparatively large number were interruptions of others, I will characterize as "dominating" – led the group into the exploration of an idea which was completely outside of the frame of the problem.

A: I guess you take the goat first. Then you leave a wolf and a cabbage...

B: But if you take the goat first you never know, because like...say if you take the goat you come back, come back to the shore, the goat, the wolf...what is it called?...

A: No.

B: Because you got the thing, you got the goat in your hand...

A: No. The goat is on the other side of the shore. You bring the goat and drop it off and come back and get the wolf.

B: But then the wolf is gonna jump out before you jump out and eat the…um, and eat the…

C: Look. You take the goat, first, what you gonna take next? Either way if you take the wolf and the goat is on the other side of the river, the wolf will eat it by the time you come back and get the cabbage.

B: So you take the wolf first, drop the wolf off…hm . . .

C: But if you take the wolf first, the goat will eat the cabbage.

B: But, listen. O.k. You can't do it. You can't.

A: Wait.

B: This is what I will do. I will take the cabbage and let them eat each other…

D: So, there is no solution.

Towards the end of this excerpt we see that student C introduced a new out-of-frame idea, which was not recognized as such, but was rejected as irrelevant. Student B then returned the group to an idea which had already been examined, and was again assessed as unacceptable. This was followed by various local assessments of the proposed ideas, in a series of interventions characterized by a lack of overall orientation and direction, and several imaginative leaps.

The exploration period went on for about four minutes. There were instances of local assessment of suggested ideas, but the episode lacked moments of global assessment of how these ideas were connected to all of the problem's conditions – apart from the negative one that the problem was insoluble. By the end of the episode, many ideas had been explored and analyzed, even some wild ones.

A: You can't do it because either way something has to be eaten.

C: Might as well let the goat eat the cabbage. Oh! Let the goat eat the cabbage,…cause you can grow another one

…………………………………………………………………………………………

C: How about this. You can bring the goat in the boat then throw the cabbage in the water,…cause the cabbage is going to float.

A: But it would float down the river. You are going across not down the river.

B: But, how about if you take the wolf across the river and drop it off. Wolf don't stand still waiting for you, wolfs run away into the wilderness, so the wolf run away and you safe with bringing others.

D: But you try to bring everybody over there.

A: Why not eat the cabbage on the way there, then you don't have to worry about the cabbage. You go pick up the goat.

B: But if you drop the cabbage off first, then the wolf will eat the goat.

A: Either way something is going to be eaten.

D: There is no solution.

It could be inferred that it was the sense of exhaustion of possibilities which led the group to look more closely at the conditions, and which triggered them to reconsider the problem and the way they perceived it. They all seemed to carry the practical assumption that the three items could be carried across the river only once, and in one direction. In addition, a great deal of energy was spent by students A and D in disciplining and monitoring students B and C – i.e. in keeping them within the framework set by the conditions of the problem.

Eventually, it was realized that the possibility of carrying things in both directions across the river had been omitted from previous assessments. At this point, student A offered an outline of a strategy. After a short negotiated analysis of the plan's compliance with the problem conditions, student A lead the group in implementing and verifying the solution.

A: Look, you can bring one over and you can bring it back if you need. Bring one over and the other back and then bring the other one over and another one back.

C: Then you could take more than three trips.

D: He has to make only three trips?

A: No, more than three trips.

D: Oh, all right.

A: Let say she brings the goat, because you can have those two over, then go back and bring the wolf and bring the goat back over, then bring the cabbage over and then go back and bring the goat. Oh, there you go. Bring the goat over first, then come back and get the wolf and then bring the wolf over then turn around and bring the goat back with you and then the cabbage and bring it with you.

D: No. It's the other way around. You bring the wolf back with you and then go get the cabbage.

A: No…cause you leave the goat with the cabbage. But listen, listen, listen. Bring the goat over, she is over here, he is over there, and then come back and get the wolf, then back over and takes the goat with her and picks up the cabbage.

D: But then you are going to have the cabbage with the goat.

A: No. You hold the cabbage in your arms. The goat ain't gonna take it from you. You leave the goat back on the other side and come back over.

C: O.K. I got you.

During the last one-minute chunk of the discussion, it was determined that every member of the group had the same understanding of the solution. Overall, the group discussion was characterized by lively, highly-motivated dynamics, although its overall tendency was to meander through the exploration of a variety of different ideas, some of which were dramatically outside of the framework conditions. There were numerous instances of control behavior on the part of the students, which was important for reaching closure of the exploration stage, and successfully terminating the problem-solving process. In short, the group went through a short episode of reading the problem, then through a long and chaotic episode of exploration, which exhibited more of a trial and error approach that any strategic exploration, and finally through a quick implementation and verification episode of the suddenly thought-out solution.

Role Analysis in Group 1: Classifying students' behaviors

Student A assumed the role of regulator. She appeared to be the most focused on the problem and its conditions. In the transcript, she is seen to have started exploring an idea immediately, but she was quickly forced into a regulator role by student B's interventions, whose ideas she recognized as out-of-frame. She continued in this mode throughout the first three quarters of the discussion, when, having more or less succeeded in "disciplining" student B, she temporarily overlooked the conditions herself for a time. But she is the student in this discussion who most consistently analyzes why suggested ideas will not work, who most often backs up her claims with arguments, and draws conclusions. She is the one who is outlining a plan (metacognitive) which is implemented at the end (metacognitive). She also kept track of each argument, and carefully monitored the final verification. Student A's behaviors were coded as reading, understanding, exploring, planning, implementing, verifying, and watching/listening (cognitive and metacognitive).

Student B took the role of instigator. She could be characterized as the element in the group that organized the system through creating resistance. The overriding frequency of her interventions and her continual prevention of the development of ideas by others are overshadowed only by her, to use Schoenfeld's term, "wild goose chase" ideas, which were dramatically out-of-frame and lacking argumentation. She appeared to lack any capacity for self-regulation. She showed no recognition of, or ability to build on, others' ideas, although she did accept, at least partially, their arguments against hers. She did show a vivid, even creative understanding of transitive logical connections, but didn't seem to be able to bring an awareness of the framework conditions to the problem-solving process in any consistent way. Her behaviors were coded as reading and exploring (cognitive).

Student C's role in the group could be characterized as reflector. She originally entered the discussion with a local assessment "Look. You take the goat, first, what you gonna take next? Either way if you take the wolf and the goat is on the other side of the river, the wolf will eat it by the time you come back and get the cabbage," and tried at first, like student A, to correct student B's thinking – pointing out, how her idea contradicted the problem conditions. From that point on, her approach to the problem and her awareness of the framework conditions was more or less identical with student B's "C: How about this. You can bring the goat in the boat then throw the cabbage in the water,...cause the cabbage is going to float." It might be speculated that her behavior was influenced by student B's thinking and behavior. Otherwise she engaged in watching/listening behaviors, without verbal indications, which is difficult to interpret. Her behaviors were coded reading, exploring, watching/listening (predominantly cognitive).

Student D assumed the role of observer, in that he spent most of the time watching/listening. At the end of the second one-minute chunk, he tried to summarize the ideas explored so far, for example: "D: So, there is no solution." Or "D: But you try to bring everybody over there." At times he appears confused, and unable to understand student A's strategy. or had difficulty articulating his thought – it is unclear whether he was suggesting a new variant of the solution or trying to restate student A's solution. Overall his behaviors were coded as reading, understanding, exploring, and watching/listening (cognitive and metacognitive).

Protocol Analysis Group 2

The protocol for Group 2 represents a discussion among three students, E, F, and G. The group read the problem without any explicit comments or statement about the conditions. They made no verbal assessment of directions that might be taken. After the short reading episode, they moved to a trial-and-error exploration. Student E began by suggesting a direction which considered one of the conditions, but overlooked the others. Student F refuted her idea by noting explicitly the overlooked condition, exhibiting a local control move. Student E suggested another out-of-frame idea, which didn't satisfy the problem conditions, but in fact became an element of discussion until the end of the conversation.

E: We can only take one at a time. So, that means we take the wolf first.
F: But the goat is gonna be left with the cabbage, so forget that one.
E: If you take the cabbage and the goat...but you can only take one at a time...
G: But if you leave one, they are gonna eat the other one.
E: Yo, you take ...,you take...You only get one at a time, you take the goat,...no, you take the wolf and you take the cabbage because the cabbage don't weigh nothing so you can put it in your pocket...
F: And then you bring the goat and then you go back and I'll take the cabbage back there. You leave the goat over there.
G: You take the wolf and the cabbage, and you put the cabbage in your pocket and you take the wolf...
E: No, you take the wolf over there and then you go back and get the goat, ...cause the goat will not eat the wolf.
F: Then take the...goat
E: The goat? They already gonna have the goat and the wolf in there...
F: ..and the cabbage
E: No, the cabbage you already have in your pocket.
F: But wait, you are still missing one thing.
E: The tiger? [laughter]
F: That's .... [inaudible]
E: No. There's only three animals.
F: We got it.
E: We got it.
G: We bring the wolf...
E: Take the cabbage – the cabbage you put in your pocket then you come back and you bring the goat-
F: ...then you go back and pull the cabbage. Oh, that was good.

This lengthy exploration without much direction appears to have been possible because no control moves – either local or global – were exercised, and no elements of the exploration were judged as irrelevant or inappropriate. Student F did try to consider the problem constraints, but after the first communicative exchange her reactions lost an apparent focus. This exploration was basically "wild" – without any goal orientation or traces of analysis, and lacking any evidence of management of the problem-solving process. The group made no attempt to verify what they arrived at as an answer, or even to make basic explanatory sense of it. They all seemed implicitly to agree upon its correctness. This group did not appear to exhibit any reflective thinking. There were several incorrect references. In short, the group moved from the short reading episode to the exploration episode and became "stuck."

Role Analysis in Group 2: Classifying Students' Behaviors

Student E's role was virtually identical to student B's (instigator) in the previous protocol, except that group resistance to her dominating behaviors did not lead to the development of a strategy, but rather ended in acquiescence. She initiated with an idea which did show awareness of the problem conditions, was returned to the problem frame by student F, and that was the first and last attempt at local control assessment. Her second out-of-frame idea was discussed briefly by the group, then accepted without any attempt to evaluate it. The behaviors of student E were coded as reading and exploring (cognitive).

Student F's role was virtually identical with that of student C above (reflector). Like the latter, she made an initial control move, but from that point on predominantly watched and listened. She did make some attempt to clarify ideas, but without substantive effect. Her behaviors were coded as reading, exploring (cognitive and metacognitive), and watching/listening.

Student G also assumed the role of a reflector. Most of her time was spent watching/listening. She didn't appear to understand the problem very well – or at least the conditions were not clear to her. Her participation consisted mostly in repeating statements made by student E. Her behaviors were coded as reading, exploring (cognitive), and watching/listening.

Protocol Analysis: Group 4

The protocol for Group 4 represents a discussion among two students, K and L. The reading episode was short and silent. Students went immediately into exploration, which started with K's out-of-frame idea. Student L refuted the idea by ignoring it, and introduced another possibility, which partially considered the problem framework, but did not coordinate all the conditions. Student K continued introducing out-of-frame ideas, with student L attempting with some indicated irritation to return him to the framework by focusing on the conditions, restating the problem, and actively monitoring the exploration process through local and state assessment. He firmly insisted on student K considering his critique.

K: I think you first have to put the cabbage in a bag and take it with the wolf to the other side

L: Wait a minute. Wait a minute. I'm thinking that you take the wolf first and when he comes back he take the cabbage, so that when he puts it over there the wolf does not eat the cabbage, so they 're left alone. They 'll do nothing. Then you come back for the goat.

K: Just throw the cabbage and . . . [inaudible]

L: It's not a part of the problem. No time to joke.

K: No. I'm serious. Throw the cabbage at a distance and the goat run away…[inaudible]

L: You can't throw the cabbage. You have to put it in the boat and cross it over the river.

K: Cabbage by itself?

L: Well. The condition is you've got to carry only one thing at a time. There's three things: a wolf, a cabbage and a goat.

In the next part, student L also tried to explore an idea which didn't satisfy all conditions, and it took him some time to realize this and abandon it. The exploration episode was focused and without major fluctuations – a result of Student L's active monitoring and local and state assessments of the process.

L: O.k. But listen, get the wolf first then you come back and get the cabbage, then you go to the other side of the river, then you throw the cabbage, then the wolf runs away, then you bring the goat to the other side, then you bring the goat and everybody is happy.

K: Why would he run away? The goal is to bring all three to the other side. You are not gonna make them run away.

L: But if you can't put the wolf and the goat together…how…

K: I'm saying…Look, you put the wolf…you cross the river with the wolf first, he stays there and then you come back for the cabbage, so the cabbage and the wolf they're alone since the wolf can't eat the cabbage.

L: The wolf does not eat cabbage,so they're left alone, so the boatman…he doesn't have to worry about anything. So then he comes back for the goat and all three of them are over there on the other side of the river.

K: But the wolf is gonna see the goat when it comes by, and then..[inaudible]

L: Is that part of the problem? Is that part of the problem?

K: But if you bring all three of them to the other side then the wolf is going to see the goat and is going to eat him. But then while the goat-

L: Stick with the problem, man. We don't want no jokes.

K: While the goat is being eaten by the wolf he's going to eat the cabbage then the whole problem is going to go down the drain.

The interesting thing here is that, the two students listen very carefully to each other's ideas. In the beginning of the episode we see that K is the one who is asking for justification of L's ideas. Very quickly the roles are reversed again.

L: I'm thinking. I'm thinking, the first thing we should do is send the goat, so that way, when it's over on the other side the wolf and the cabbage they're alone, so there's nothing wrong there. But then from here on there is a problem right there, because if you send the cabbage, the goat will eat it, and if you send the wolf over there, then--

K: O.K. But go down the river then--

L: Send it across, not to go down the river.

K: I'm saying more down so they don't see each other.

L: I'm thinking, there is no solution to this.[pause]

K: I'm saying what I think…it would make more sense if the…I mean, come on, a wolf in a boat, it's gonna kill the guy, it would probably kill me. Because I'm trying to tame the wolf and then the goat would probably eat my shoes and the cabbage…you know…

L: He will have his own protection from the wolf. Don't worry.

K: Are we just trying to get all three of them to the other side. That's it?

L: That's it. That's the goal.

K: So you just bring one, then one, then one. Bring the goat first, then the cabbage.

L: No, you got to understand what's the situation—what's going on. If you send one, the two on the other side, they are gonna, a-a ….I say if you send the wolf in first the goat and the cabbage, they're left alone. But the goat is gonna eat the cabbage. If you send the cabbage over, then the wolf will eat the goat.

K: So, send the goat over…

L: Yah, that's what I'm thinking. That's the first step, it's, um,… just have no choice. I mean either…, um,… send either one of them and something goes on.

K: So send the goat, then take the wolf, throw him in the river, then take the cabbage and then when the goat tries to eat the cabbage, just run away. That could work.

The planning episode was overt and well articulated, but it dealt with only some of the problem's condition.

Student L assessed what he considered necessary to do to move forward, identified the source of the conflicts that were preventing a solution, and then moved to a short analysis episode. Here student K followed up by moving the process into another exploration episode by introducing new out-of-frame ideas. This particular episode lacked any assessment on the part of the participants regarding the success or the failure of their problem-solving process. Student L was trying to help student K understand the conditions, but at the same time he himself was drawn to ignore the conditions imposed by the problem.

L: But does it have to be on a boat or I mean, uh... You said that the boat man can only carry one thing at a time. So how about taking one on the boat and the other just dragging in through the river. Can we do that? [Asking the teacher?]

Teacher: No.

[pause]

L: Why don't you just keep a fence on the other side of him, just keep the goat on one side and the wolf on the other?

K: The goat will eat the metal and in the fence and the wolf will just jump over it and eat the goat and the goat will eat the cabbage.

L: Are they like running free or something like...is there something like a cage? [A question to the teacher again.]

Teacher: No. I don't think so.

L: All right. I'm just curious. I was thinking if you send both of them like the wolf and the goat over there, what's the point of worrying, because the wolf is in the cage. I mean, he can't get in.

K: Send the wolf then send the cabbage, then go down the river, and send the goat.

L: [inaudible]...the wolf, but then when you get there you put in the goat in the boat, if you take him back and then drop him off and bring the cabbage in, put them over there so the wolf and the cabbage, they're alone. Go back and get the cabbage. I got it. [sotto voce]. That's it.

K: What?

L: All right. This is how it is. [drawing] Here is the river and here is where all three of them are, and here is where they're supposed to be. [explanations continue]

At this point he used the teacher as referent, asking her questions about the relevance of his ideas, thus expressing the need for some external assessment. The teacher may have helped him here to abandon these ideas and to look for a strategy which would satisfy the whole set of conditions. Then apparently he "saw" a strategy and came up with a relevant plan, which was quickly followed by implementation. Student L was clearly confident with the solution, and the verification episode went as quickly: after brief initial resistance, student K agreed on L's solution.

Role Analysis in Group 4: Classifying Students' Behaviors

Within the context of the dyad, Student K could be characterized as a focus distractor –although, as with students B, E, and I above, it could be hypothesized that he created the resistance necessary for the process to self-organize (this is more difficult to claim with a dyadic situation). He consistently resisted the many efforts by student L to persuade him to frame his exploration ideas within the given conditions. He appears to generally exhibit unreflective thinking. It wasn't clear at the end whether he understood the solution. His behaviors were coded as reading, exploring (cognitive)

Student L played a role of a regulator. He spent a great deal of time explaining the conditions, reformulating the problem, and trying to keep student K within the framework. He worked single-handedly to monitor and evaluate the process – attempting to orchestrate, and exhibiting conscious local, global and state assessment and time management behaviors. He was trying to orchestrate the process, exhibiting assessment and time management. His regulatory functions were verbalized in statements like "Stick with the problem," or "It's not a part of the problem. No time for jokes." His behaviors were coded as reading, understanding (metacognitive), exploring (cognitive and metacognitive), planning (metacognitive), analyzing (metacognitive), implementing (metacognitive), and verifying (metacognitive).

Below is a table that represents the individual students' cognitive and metacognitive behaviors in percentages, based on the analysis of the discussions.

Groups	Categories Students	Metacognitive	Cognitive	Total %
Group 1	A	25.4	7.2	32.6
	B	0	14.5	14.5
	C	12.7	5.4	18.1
	D	9	3.6	12.6
Group 2	E	0	33.3	33.3
	F	6.6	6.6	13.2
	G	6.6	6.6	13.2
Group 3	H	33.3	11.2	44.4
	I	0	22.2	22.2
	J	18.5	7.4	25.9
Group 4	K	0	34.4	34.4
	L	50	15.6	65.6
Group 5	M	0	33.3	33.3
	N	0	33.3	33.3
	O	0	22.2	22.2

Table 1: Table of the % of cognitive and metacognitive total behavior per student

Results and Discussion

Protocol analyses were carried out using Schoenfeld's framework for macroscopic analysis. They focused mostly on monitoring, control and regulatory functions of the thought processes, and on tracing the consequences according to whether these behaviors were present or absent. Each session was parsed into episodes. Junctures between episodes delineated points at which major shifts in resource allocation and direction of the problem-solving process were executed. The analyses demonstrate that absence of monitoring and assessment at the control level can generate failure in problem solving. Of the five groups examined, groups 1, 3 and 4 were successful with the problem solving and groups 2 and 5 were not. Groups 1, 3 and 4 exhibited a fair amount of reasonable control decisions – evaluating and curtailing a number of possible approaches while working on the problem, and spending a relatively limited amount of time on "wild goose chases." This allowed for the emergence of new possibilities for solving the problem.

The overall quality of the students' monitoring, assessment and executive decision-making in these groups was relatively poor. In many cases they made detours, and pursued out-of-frame ideas. Group 1, for example, spent a comparatively long time during the exploration episode to pursue several out-of-frame ideas, but their monitoring, assessment and executive skills made positive contributions to a successful problem-solving performance nevertheless. Groups 2 and 5 explored several out-of-frame ideas without pausing to consider the problem constraints, and consequently failed to discard these ideas. The absences of monitoring and control behaviors were perhaps the major contributing factors to these groups' failures.

The episode analyses provide a clear contrast between the two sets of groups. Groups 1, 3 and 4 are much more dynamic and rich in episodes and transitions. The analyses of the transcripts of Groups 2 and 5 indicate only reading and exploration episodes, with no transitions. There were no evidences of shifts of direction, which precluded the possibility of finding a solution. All this suggests that monitoring and control decisions can be crucially important for the success of problem solving. It suggests that education with an emphasis on metacognition can have a significant effect on students' behaviors at the control level, and therefore positive effect on problem-solving performance.

Coding of students' cognitive behaviors was carried out using Artzt & Thomas's (1993) framework. It delineates both individual students' and groups' profiles. Figure 1 represents the percentage of behaviors coded as metacognitive, cognitive and watching/listening. The total percentage of the metacognitive behaviors of the students varies between 0% and 50%. The range of the individual cognitive behaviors as a percentage of the total behaviors coded ranged from 3.3% to 34.4%. The total percentage of metacognitive behaviors exhibited in the groups varies between 0% and 51.8%. The range of the cognitive behaviors in the groups' profiles is between 30.4% and 58.8%.

A close reading of the transcripts and their analyses indicates other factors which played a role in the incidence of executive and control behaviors in the problem-solving process. I would like to note two of them. One of them is a discursive clash between the subjects and the teacher – i.e. a conflict between the academic discourse of classroom mathematics and of the students – which was especially evident in a word problem which did not involve number calculation. There are many examples in these transcripts of cases in which the students were either unable or did not care to restrict their thinking to within the frame defined by the conditions of the problem, which is the major element of mathematical discourse.

Another factor which appeared to play a significant role in these particular discussions is the degree of tolerance of others' opinions manifested by the students, and by the distribution of power within the group system. In Group 1 for example, student B played a dominant role, which was partially neutralized by the interventions of students A and C, who were monitoring and trying to exercise some control over the process. Student D was blocked and not given chance to participate at all. Near the middle of the discussion Student B was called away by the teacher for another class, and the group began functioning more evenly – communication patterns changed and allowed for the emergence of dialogue between all of the members. The resulting exchange of ideas and the addition of an analytic dimension to the exploration provided a new opportunity for success.

Group 5 showed a dysfunction of another kind. All three members were completely undirected in terms of goal, and jumped from one idea to another, without any instances of assessment or review. No attempt was made to understand each other's ideas, or collaborative work on ideas. This could be characterized as a situation of goal displacement. The participants gave the impression of being motivated primarily by a desire both to impose their opinions on others and to reject the opinions of others, which implies a double-bind – at least in the world of mathematical discourse. As a result, the discussion exhibited a dramatic lack of cohesion. In Group 3 one student (I) was unfocused and two students (H and J) were making a significant effort to bring her into focus, thus balancing the system such that it remained relatively goal directed, which was important for its success. In Group 4 – the only dyadic group – two students were competing for domination. One of them (K) had the disadvantage, not only of lacking focus, but of appearing to be unfamiliar with the constraint structure of mathematical discourse, while the other (L) was perhaps the most aware of all the students in the experiment of the discursive demands of the problem. L used this power to assert control of the process, but it could also be suggested that their standoff contributed to a relative balance in the system, which allowed for the possibility of success. In summary, this sort of analysis of the group dynamics of problem-solving groups yields the possibility of a connection between the degree of tolerance and the distribution of power within the system on the one hand, and metacognitive knowledge, control execution in the system and successful problem-solving on the other.

Comparison of the groups' profiles supports these conclusions on a micro level. They suggest that the groups with the lowest total percentage of metacognitive behaviors (Groups 2 and 5) were the most unsuccessful in terms of the problem solving task. And it is worth noting that the individuals with the highest percentage of metacognitive behaviors were students L, H and A, who were the most active participants and who exhibited the greatest self-control. They were the plan-makers and the energetic collaborators in the implementation and verification episodes.

Another distinction between the successful and unsuccessful groups is that the metacognitive and cognitive behaviors in the first set of groups were fairly well-balanced – 47.1% vs. 30.4% in Group 1; 57.8% vs. 40.7% in Group 3; and 50% vs. 50% in Group 4 – whereas the imbalance in the second set of groups is obvious (13.2% vs. 46.5% in Group 2, and 0% vs. 58.8% in Group 5). Structural analysis thus indicates that groups that functioned well in terms of equal participation and involvement of all of the members were more successful than groups in which there were larger variations. This would suggest that a well-functioning collaborative problem solving group requires a balance between the cognitive and metacognitive knowledge of the participants.

The foregoing analyses support the idea that the relatively equal participation of all members of a collaborative group, without pronounced patterns of domination, and with a high degree of tolerance to others' opinions, are factors that contribute to an optimal environment for well-functioning and successful cooperative work in group settings. This research also seems to indicate that a balance of cognitive and metacognitive moves creates a greater possibility than otherwise for monitoring and regulation to occur during collaborative problem solving. It also seems to support Schoenfeld's (1989) contention that in collaborative problem-solving activities, achieving the solution to a problem becomes secondary to negotiating a shared problem definition and a common means of communication. In this case, groups with weak communicative patterns were less successful in problem solving than the groups which functioned better as a communicative system. Negotiation of a shared problem definition in many cases turned out to trigger reformulation of the problem in another frame.

At least two questions emerged in the course of this study which could be investigated further: 1) How great a role do the factors mentioned above play in collaborative work? and 2) Could there be other factors of equal or even greater importance that influence this process?

Conclusion

The purpose of this study was to examine the role of cognition and metacognition in small group settings. Two frameworks were used to delineate the types and levels of thought processes and their interrelationships within existing classroom culture. The analysis suggests that different processes serve specific functions, and that the interplay between and among them accounts for problem solving effectiveness. This study suggests that these frameworks may be useful for action research in the classroom, and they promise to be a feasible tool for studying classroom cognitive dynamics.

It goes without saying that a teacher needs some understanding of classroom cognitive dynamics in order to be able to adapt her practice to different classroom contexts. If the teacher knows what students are capable of doing on their own, she can take them to a more difficult level at the appropriate moment, assist them in the active construction of new knowledge, and thus enhance if not accelerate cognitive development. In addition, the regular practice of collaborative problem-solving promises to render students' thinking processes more explicit, thereby exposing those processes to more immediate intervention on the part of teachers and peers, and stimulating both group and individual self-correction.

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