MATH 260: Introduction to Abstract Mathematics

Spring 2011, 4 credits, MWF 9:00-9:50 am, R 8:00-8:50 am, MRC 444
Instructor: Richard J. Maresh, Associate Professor of Mathematics
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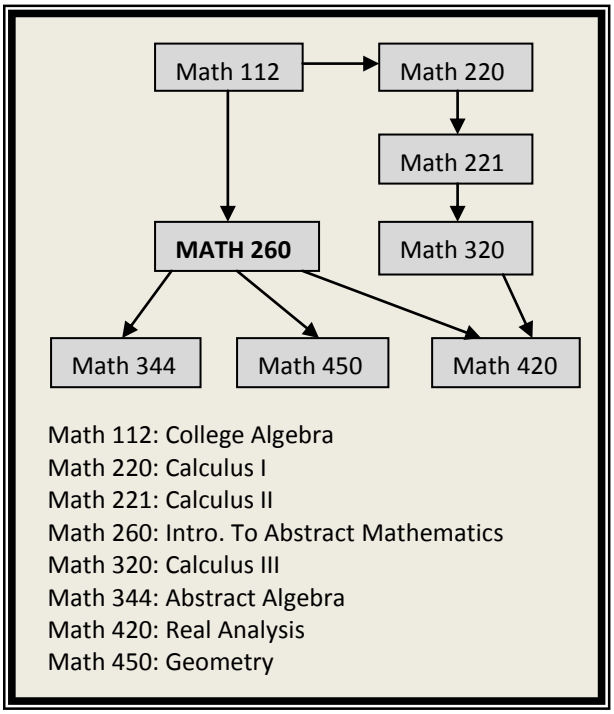
Final Exam: Thursday, 12 May 2011, 7:40-9:40 am

$$\forall \varepsilon > 0,\ \exists\, \delta > 0 \ni |x-a| < \delta \Rightarrow |f(x)-L| < \varepsilon$$

Course Description: Sentential and quantifier logic, axiomatic systems, and set theory. Emphasis is on the development of mathematical proofs. Pre-requisite: grade of C or higher in Math 112.
(Note: This course is a prerequisite for MATH 344, 420, and 450, all of which require the ability to write proofs.)

Textbook: *An Introduction to Abstract Mathematics*, by Robert Bond and William Keane. (Waveland Press, 2007)



Math 260 in the context of the Mathematics major:
The diagram at the right spells out the role of this course within the context of the mathematics major. Math 260 is the course in which majors are introduced in a serious way to the craft of logic and proof. The "scientific method" is experimentation, and the scientist must learn to develop hypotheses and then to create ways to test these hypotheses. The "mathematical method" includes the same sort of inductive reasoning to develop hypotheses, but these hypotheses are proved or disproved by logic, by deductive reasoning, by *PROOF*. Because logic and proof are the heart and soul of mathematics, Math 260 is a pre-requisite for a number of courses in the final tier of courses in the major. The calculus sequence is also important, of course, but it largely plays the role of applied mathematics in the major. Math 260 will give you an indication of what mathematics is really about, and about whether or not you have what it takes, and whether or not you feel the passion necessary to be a mathematician! Mathematicians refer to a course like MATH 260 as a "bridge course", in that it takes students from their high school algebra and pre-calculus mathematics to readiness for the study of more advanced mathematics. Most students find it challenging, but also very rewarding; success in 260 is a good indicator of success in subsequent courses in the major.

Assessment of the Mathematics Major:
It is important that each of our majors develop the abilities and understanding necessary for the mathematical practice, whether as a teacher or as a working mathematician in some applied situation. The content is fairly obvious from the course titles and syllabi, but there is also a set of desired outcomes that the mathematics program seeks to assess as we have students work their way through the program. In essence, we want students to be able to reason logically, to solve problems, to communicate their solutions and proofs, and to appropriately use technological tools. The specific outcomes are listed here:

Viterbo University Student Learning Outcomes (SLO) For the Mathematics Major:

SLO 1: Our majors will be able to reason inductively. (Reasoning)
SLO 2: Our majors will be able to reason deductively. (Reasoning)
SLO 3: Our majors will demonstrate the ability to apply appropriate mathematical methods to novel or non-routine problems. (Problem Solving)

SLO 4: Our majors will demonstrate the ability to perform computational, graphical and algebraic procedures using a calculator or computer. (Technology)
SLO 5: Our majors will use the language of mathematics accurately and appropriately in oral presentation. (Communication)
SLO 6: Our majors will use the language of mathematics accurately and appropriate in written form. (Communication)

We have developed a system for assessing these outcomes at appropriate places throughout the program. At the end of each course in the major, we assess how well our students have demonstrated the various outcomes the specific course addresses. This assessment system is not formally used in determining student grades – in fact, the assessment ratings reflect the grades the students have earned. This system is summarized in the grid below:

	220	221	230	260	320	321	330	340	344	365	420	450	499
SLO 1				X			X	X	X		X	X	
SLO 2	X	X	X	X	X	X	X	X	X		X	X	
SLO3	X	X	X	X	X	X	X	X	X		X	X	
SLO 4	X	X	X		X	X	X	X					
SLO 5				X			X	X	X		X	X	X
SLO 6	X	X	X	X	X	X	X	X	X		X	X	X

Course Goals:
To put it in a more specific way, these are the goals for this course:
1. The over-arching goal in this course is to learn to read and write mathematical proofs.
2. A second major goal is to be able to communicate mathematical concepts and mathematical proofs, using appropriate mathematical notation and language, both in writing and orally.
3. While this course is more about process than content, it is still a goal of this course to consider content which is important to further study of mathematics: logic, set theory, relations and functions, mathematical induction, the real number system.

Course Outline:
From the perspective of content, we will cover the following topics (essentially the first five chapters of the text):
1. Mathematical Reasoning: Statements, Implication, Proof
2. Sets: Sets, Subsets, Set Operations
3. Functions: Definition, One-to-one and Onto Functions, Composition of Functions
4. Binary Operations: Binary Operations, Equivalence Relations
5. The Integers: Axioms, Induction, Greatest Common Divisor, Primes, Factorization, Congruence

Course Procedures:
This course is a very significant step for every mathematics major, and probably more than any other single course, certainly up to this point in your course work, lets you see what it is like to be a mathematician and whether you can find fulfillment as, or have the ability to be, a mathematician. I have vivid memories of my own equivalent course as a student!
All mathematics educators believe in the adage, “Mathematics is not a spectator sport!” In this course, more than in most, however, your contributions will MAKE the course. I intend to allow a healthy fraction of our class time to consist of discussion about, and student presentations of, attempted proofs. In addition to learning to read or listen to proofs, with comprehension, as well as learning to problem solve and write proofs, it is important to acquire a disinterested objectivity - like a scientist, we must strive to view a proof in its own terms and not take corrections personally. The “craft” of mathematics is in one sense very personal but at the same time your work must be held to the highest standard of objectivity.
I have chosen this textbook largely for its philosophy; it is designed to be used in a student-oriented class. I will make assignments virtually on a daily basis and it is very important that you work on these as best you can. I hope that at least once a week, on average, each of will get a chance to present some of your work. It is important that

you not miss classes; this will be almost a “seminar” course - the size of the group and the philosophy will allow us to do that.
As we work our way through the course, I will occasionally collect homework problems and grade them. It is important that I can see how well you are coping with the material and it is important that you get regular feedback about your success.

Grading: I will use the traditional 90-80-70-60 scale as a framework for assigning grades. I do realize the artificiality of such a scale - but I usually try to make the assignments reasonable enough so that students can earn an appropriate grade. It is harder in this course in particular to make things seem “objective”, since writing proofs is something like writing essays in an English course - there is “correct” and “complete”, but there is also “elegant” and “insightful”. I will try to keep you informed along the way about your progress in the course.

Americans with Disability Act: If you are a person with a disability and require any auxiliary aids, services or other accommodations for this class, please see me or Jane Eddy, the disabilities (ADA) coordinator (Learning Center, 796-3194) within ten days to discuss your accommodation needs.

One note about the schedule – it is difficult to pin down a daily schedule for a course like this one. The following gives a general framework but I may have to make little adjustments along the way, depending on how things are going. Still, I want to indicate a content outline and a pace I hope will be reasonable.

MATH 260 Schedule, Spring 2011

17 Jan	[1.1] Statements, Quantifiers	p 12 # 1-5
18 Jan	[1.1] Proof	p 13 # 6-11
19 Jan	[1.1] more…	p 14 # 12-13, D1-3
21 Jan	[1.1] more …	p 14 # D5-D9
24 Jan	[1.2] Compound Statements: Conjunctions, Disjunctions, Negations	p 26 # 2-8
25 Jan	[1.2] Logical Equivalence, Tautologies, Contradictions	p 26 # 11, 13, 14, D1-D5
26 Jan	[1.3] Implication: If-Then Statements, Proofs, Counterexamples	p 35 # 1-7
28 Jan	[1.3] Necessary and Sufficient Conditions	p 36 # 8-10, 12-18
31 Jan	[1.3] more …	p 37 # D1-D4
1 Feb	[1.4] Contrapositive and Converse	p 44 # 2-6, 8-10, 11-13
2 Feb	[1.4] Biconditionals, Proof by Contradiction	p 45 # 14-18, 20-21
4 Feb	[1.4] more …	p 45 # D1-D5
7 Feb	[2.1] Sets and Subsets	p 57 # 1-6, 8-10
8 Feb	[2.1] Complements	p 59 # 12-17, 19-22
9 Feb	[2.1] more …	p 60 # D1-D5
11 Feb	[2.2] Set Operations: Unions, Intersections	p 68 # 1-9
14 Feb	[2.2] DeMorgan’s Laws, Cartesian Products	p 69 # 10-18, 22-25
15 Feb	[2.2] more …	p 71 # 28, 29, D1
16 Feb	[2.3] Collections of Sets: Power Sets	p 78 # 2-8, 10
18 Feb	[2.3] Partitions, Pigeonhole Principle	p 80 # 20-23, D1, D2
21 Feb	Review …	
22 Feb	**EXAM #1**	
23 Feb	Exam review	
25 Feb	[3.1] Functions: Definitions and Basic Properties	p 93 # 1-6
28 Feb	[3.1] Image of a function	p 94 # 8-12, 15-19
1 Mar	[3.2] Injections, Surjections, Bijections	p 105 # 1-3, 5-7
2 Mar	[3.2] more …	p 106 # 8-13, 16-18
4 Mar	[3.2] more …	p 108 # 20-23, 26, 28, 29, D7

7-11 Mar	*-- Spring Break --*	
14 Mar	[3.3] Function Composition	p 118 # 1-3, 5, 7-9
15 Mar	[3.3] Function Inverses	p 119 # 10-17, D2
16 Mar	[4.1] Binary Operations, Properties	p 134 # 1-7, 13-17
18 Mar	[4.1] more …	p 135 # 18-20, 29-33
21 Mar	[4.1] more …	p 137 # 34, D1, D2, D5
22 Mar	[4.2] Equivalence Relations	p 147 # 1-11
23 Mar	[4.2] Equivalence Classes, Partial and Linear Orderings	p 149 # 12-17, D1, D2
25 Mar	Review …	
28 Mar	**EXAM #2**	
29 Mar	Exam Review	
30 Mar	[5.1] The Integers: Axioms, Basic Properties	p 157 # 1-9
1 Apr	[5.1] Well-ordering Principle	p 157 # 10-18
4 Apr	[5.1] more …	p 158 # 20, 24, D1-D3
5 Apr	[5.2] Mathematical Induction	p 169 # 1-6
6 Apr	[5.2] more …	p 169 # 7-9, 12-13, 24-25
8 Apr	[5.2] more …	p 173 # D2-D4
11 Apr	[5.3] The Division Algorithm	p 180 # 1-5, 7-8, 10-11
12 Apr	[5.3] Greatest Common Divisor, Relatively Prime Integers	p 180 # 12-16, 20-22, D2, D3, D5
13 Apr	[5.4] Primes and Unique Factorization	p 186 # 2, 3, 6-12
15 Apr	[5.4] more …	p 187 # 13-15, 17-20, 24, D1-D3
18 Apr	[5.5] Congruence	p 197 # 1-8, 11
19 Apr	[5.5] more…	p 197 # 12-13, 15-20
20 Apr	[5.5] more …	p 198 # 21-27, 30, D1
21-25 Apr	*-- Easter Break --*	
26 Apr	[5.6] Generalizing a Theorem	p 206 # 1, 2, 4-6
27 Apr	[6.1] Countability	p 218 # 1-5
29 Apr	Review…	
2 May	**EXAM #3**	
3 May	Exam Review	
4 May	Review for Final Exam	
6 May	**Practice Final Exam** (25 points)	

FINAL EXAM: Thursday, 12 May 2010, 7:40-9:40 a.m., 125 points